

CANONICAL form :-

A Boolean Function expressed as Sum of minterms or product of maxterms are said to be Canonical form.

SOP and POS are said standard form also

$$F = AB + CD$$

$$F = (A+B)(C+D)$$

$$F = (AB+CD)(\bar{A}\bar{B} + \bar{C}\bar{D}) \quad \text{non standard form}$$

$$\text{Ex)} \quad F = \underbrace{\bar{A}\bar{B}\bar{C}}_{m_0} + \underbrace{AB\bar{C}}_{m_6} + \underbrace{A\bar{B}C}_{m_5}$$

$$F = \sum 0, 5, 6$$

$$\text{Ex)} \quad \text{Put } F \text{ in SOP, } F = \sum 1, 3, 4, 7$$

$$\text{Sol}_1 - \quad \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + ABC$$

Ex) change to (expanded form)

$$F = AB + ABC + \bar{A}\bar{B}$$

Note:- Represent the Boolean function in (SOP), all the variables must appear in the function.

$$\text{Sol}_2 - \quad F = AB(C + \bar{C}) + ABC + \bar{A}\bar{B}(C + \bar{C})$$

$$= \underbrace{ABC}_{7} + \underbrace{AB\bar{C}}_{6} + \underbrace{\bar{A}\bar{B}C}_{1} + \underbrace{\bar{A}\bar{B}\bar{C}}_{0}$$

$$F = \sum 0, 1, 6, 7$$

$$\text{Ex)} F = A + BC$$

$$\begin{aligned} \text{Sol}^{\wedge}:- F &= A(B + \bar{B})(C + \bar{C}) + BC(A + \bar{A}) \\ &= (AB + A\bar{B})(C + \bar{C}) + ABC + \bar{A}BC \\ &= \underset{7}{ABC} + \underset{6}{A\bar{B}\bar{C}} + \underset{5}{A\bar{B}C} + \underset{4}{A\bar{B}\bar{C}} + \underset{3}{\bar{A}BC} \\ &= \end{aligned}$$

$$\therefore F = \sum 3, 4, 5, 6, 7$$

Ex) Put (F) in POS

$$\begin{aligned} F &= (A + B)(\bar{A} + C)(A + \bar{B} + C) \\ F &= \underset{0}{(A + B + C)} \cdot \underset{1}{(A + B + \bar{C})} \cdot \underset{2}{(\bar{A} + C + B)} \cdot \underset{4}{(\bar{A} + C + \bar{B})} \cdot \underset{5}{(A + \bar{B} + C)} \end{aligned}$$

$$\therefore F = \pi 0, 1, 2, 4, 5$$

Ex) Represent F_1 & F_2 by means of using (SOP) & (POS)

Sol[^]:-

SOP

$$F_1 = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$F_1 = \sum 3, 5, 6, 7$$

$$F_2 = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$$

$$F_2 = \sum 0, 1, 5, 7$$

POS

$$F_1 = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + C)$$

$$F_1 = \pi 0, 1, 2, 4$$

$$F_2 = (A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})$$

$$F_2 = \pi 2, 3, 4, 6$$

A	B	C	F_1	F_2
0	0	0	0	1
0	0	1	0	1
0	1	0	0	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

Simplification of Boolean Functions

There are two types of minimization:—

- ① Karnaugh Map (K-Map).
- ② Tabular Method.

① Karnaugh Map (K-Map):—

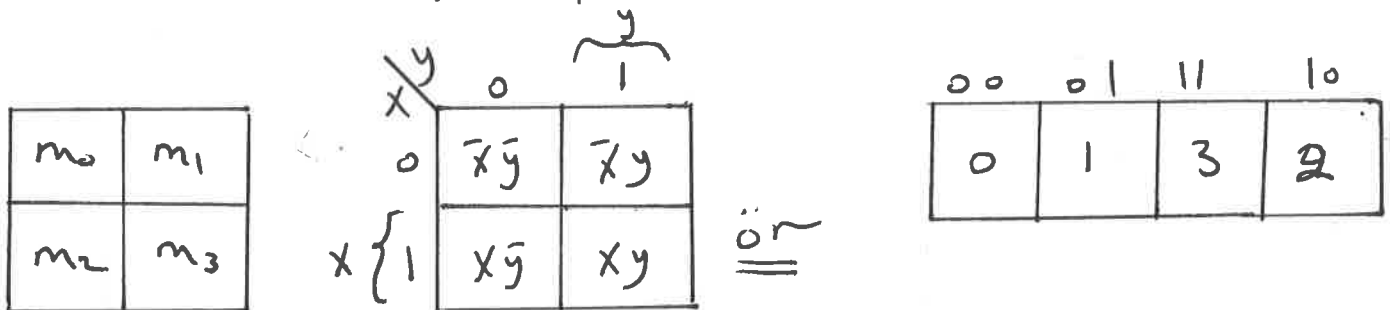
Are used to simplify the Boolean expression, it consist of neighbour cells that take the square form. The number of squares dependant on the number of variables, if the number of variables (n) then number of square (N)

$$N = 2^n$$

The map is a diagram made up of squares, each square represents one minterm.

① In case of two variables

There are four minterm for two variables: Hence the map consists of four squares, one for each minterm.



Two - variables Map

② In case of 3-variables:—

There are eight minterms for three binary variables, therefore a map consist of eight squares.

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

		y			
	z	00	01	11	10
x	0	$\bar{x}\bar{y}\bar{z}$	$\bar{x}y\bar{z}$	$\bar{x}yz$	$\bar{x}y\bar{z}$
	1	$x\bar{y}\bar{z}$	$x\bar{y}z$	xyz	$xy\bar{z}$

Three-Variables Map

Ex) Simplify the Boolean Function?

$$F = \bar{x}yz + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z}$$

Solⁿ—

		y			
	z	00	01	11	10
x	0			1	1
	1	1	1		

OR

		x	
	y	0	1
z	00	$\bar{x}\bar{y}\bar{z}$	$x\bar{y}\bar{z}$
	01	$\bar{x}y\bar{z}$	$x\bar{y}z$
	11	$\bar{x}yz$	xyz
	10	$\bar{x}y\bar{z}$	$xy\bar{z}$

$$F = x\bar{y} + \bar{x}y$$

Ex) Simplify the Boolean Function?

$$F(x,y,z) = \sum (2, 3, 4, 5, 6)$$

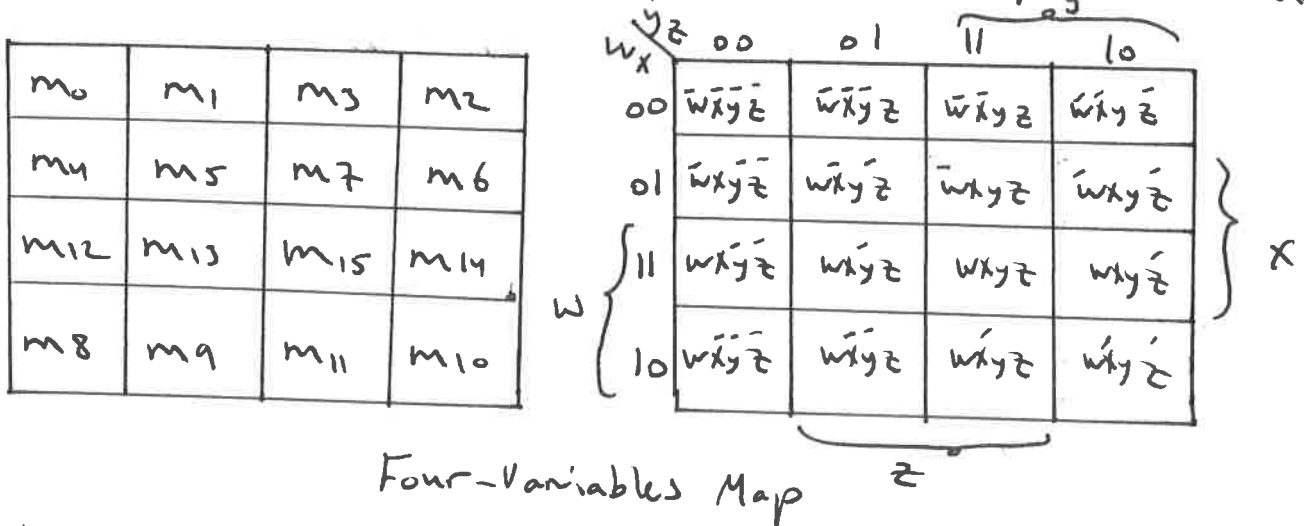
		B			
	C	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
A	\bar{A}	1			1
	A	1	1		

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$F = \bar{C} + AB$$

③ Four-Variables Map

It consist of 16 minterms, therefore 16 square are needed.



Note:-

one square represents one minterm, giving a term of four literals.

* Two adjacent square represent a term of three literals.

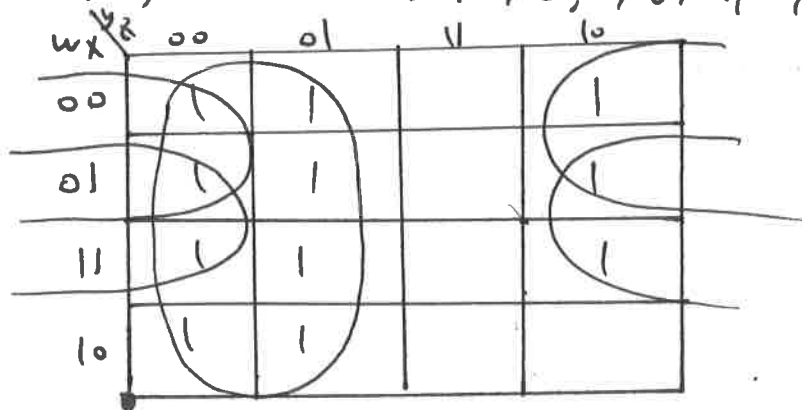
* Four = = = = two = .

* Eight = = = = one = .

* Sixteen = = = = equal to 1.

Ex) Simplify the Boolean Function?

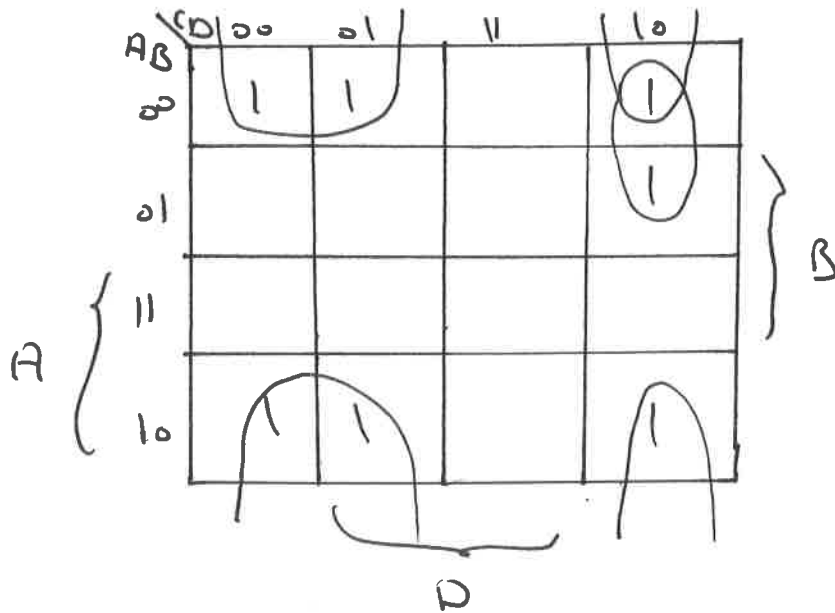
$$F(w, x, y, z) = \sum (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$



$$F = \bar{y} + \bar{w}\bar{z} + x\bar{z}$$

Ex) Simplify the Boolean function

$$F = \bar{A}\bar{B}\bar{C} + \bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}\bar{C}$$



$$F = \bar{B}\bar{C} + \bar{A}C\bar{D} + \bar{B}C\bar{D}$$

④ Five- and Six-Variables Maps: -

AB	CDE							
	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

E
D
E

Five-Variable -Map

Note:- In some cases in number of (0's) more than (1's) we can take the (0's) instead of (1's), but the function will be \bar{F} not F .